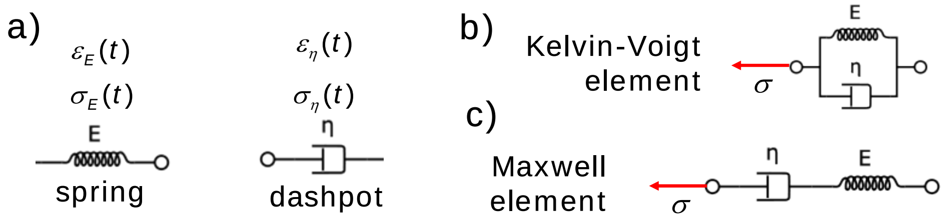

Figure 1: a) Pictograms used for a spring and dashpot, representing elastic and viscous elements, respectively. The corresponding quantities characterizing the time-dependent stress and strain state of the element are shown above. b) A Kelvin-Voigt element. c) A Maxwell element.

Problem set 3, due on October 6, **before** the beginning of the lecture

1 Viscoelastic response and analogy to electrical circuits (group exercise, 10 points)

Solve this exercise in groups of three to four students. You are allowed to submit a computer printout (e.g. created using Word, LaTeX etc.).

In class we discussed the viscoelastic behavior of polymers and modeled the elastic component with a spring obeying Hooke's law and the viscous component with a dashpot.

a) It was mentioned that there is a certain analogy between the physical laws governing viscoelastic behavior and the laws in an electrical circuit (EC) consisting of a network of resistors, capacitors and inductors. Here we want to explore how far this analogy actually goes. Looking at the relations between stress and strain for the two mechanical elements spring and dashpot, which of the three electrical circuit components do they correspond to? Which physical quantity in the electrical circuit does the stress σ correspond to (the symbol A will be used for the EC equivalent below). What about the strain ϵ (the EC equivalent will be called B)?

Having established these analogies, which EC configuration (parallel connection, serial connection?) would you intuitively expect to correspond to the (combined) Maxwell element and the (combined) Kelvin-Voigt element, respectively? Let us check if this intuitive expectation is correct: Write down how the total σ and ϵ are composed of σ_E, σ_η, ϵ_E and ϵ_η for the Maxwell and the Kelvin-Voigt elements, respectively. Compare this to how the total A and B are composed of the quantities A_X, B_X, A_Y and B_Y for a parallel connection and a serial connection in an EC circuit. What is your conclusion?

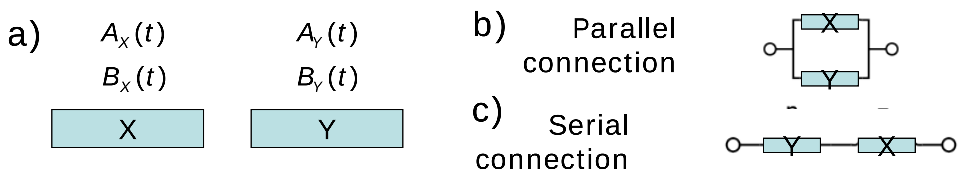

Figure 2: a) Electrical circuit elements to be determined by analogy to a mechanical spring and a dashpot. The corresponding time dependent physical quantities A, B, to be determined by analogy to the mechanical case as well, are shown above. b) Parallel EC connection. c) Serial EC connection.

b) Derive and sketch the time dependence of the total $\epsilon(t)$ for a Maxwell element in a creep experiment (i.e. the total stress changes from 0 to a constant value σ_0 at $t = 0$). Is a creep experiment also possible for a Kelvin-Voigt element(justify your answer)? If yes, derive the time dependence of $\epsilon(t)$ for this case as well.

c) Derive and sketch the time dependence of $\sigma(t)$ for a Maxwell element in a stress relaxation experiment (the strain changes from 0 to a constant value ϵ_0 at $t = 0$). Is a stress relaxation experiment also possible for a Kelvin-Voigt element(justify your answer)? If yes, derive the time dependence of $\epsilon(t)$ for this case as well.

d) Derive and sketch the time dependence of $\epsilon(t)$ for a Maxwell element in a dynamic experiment with an oscillating stress $\sigma(t) = \sigma_0 \exp(-i\omega t)$. Hint: Do you expect the system to oscillate with the same frequency as the applied stress? With which amplitude? Will there be any time shift in the oscillations with which the system responses to the stress (if yes, derive it)?