Learning goals for today--
Develop intuitive understanding for relationship between shapes of wave functions and position, momentum, and energies of the particles they describe.

1. “What is psi?”
superposition and interference of waves

2. Relating shape of wave function to momentum and kinetic energy of particle. practice

3. Heisenberg uncertainty principle-- use of.

4. 12:00 discussion of student feedback--
changes being made in response

Reading quiz

1. For a wavepacket $\delta f\ \delta t$ ~

a. h   b. $\hbar$ c. 1   d. 0

2. Heisenberg uncertainty principle as expressed in the book is

a. $\Delta x \Delta p \geq 1$ b. $\Delta x \Delta t \geq 1$ c. $\Delta x / \Delta p \geq h/2$ d. $\Delta x \Delta p \geq h/2$

3. The Heisenberg U. P. is

a. an inherent implication of the wavelike nature of particles.

b. is an additional new concept beyond the wavelike nature of particles.

c. this issue is not discussed in the textbook.

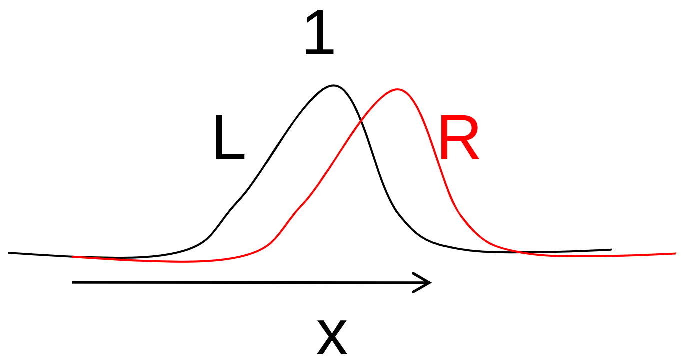
Friday tutorial -- solve Schrod. eq. for easiest (but important!) cases. (like HW) 1. Electron in free space.

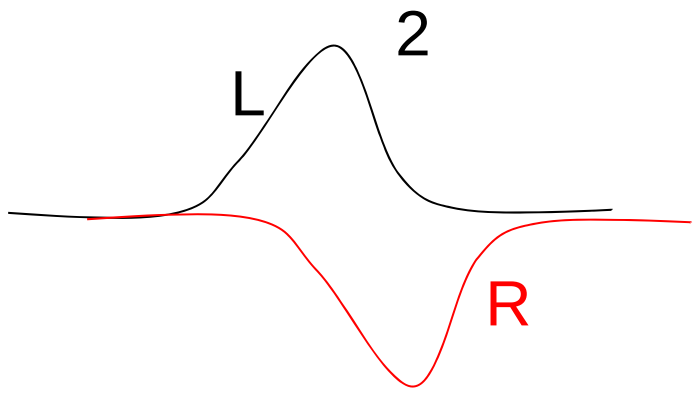
today and wed-- talk about some implications, set up issues involved in more general cases.

“How small does object have to be for nano to become special in nanotechnology? i.e. quantum mechanics becomes important.”

good question from multiple students-- “What is psi?”

we always talk about $|\psi|^2$ is probability, and what we measure. Can we just ignore thinking about psi and only keep track of the probability?

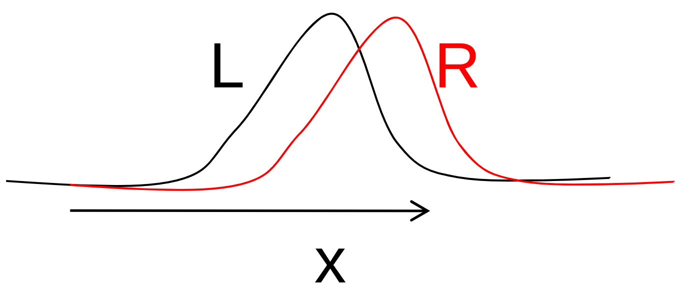


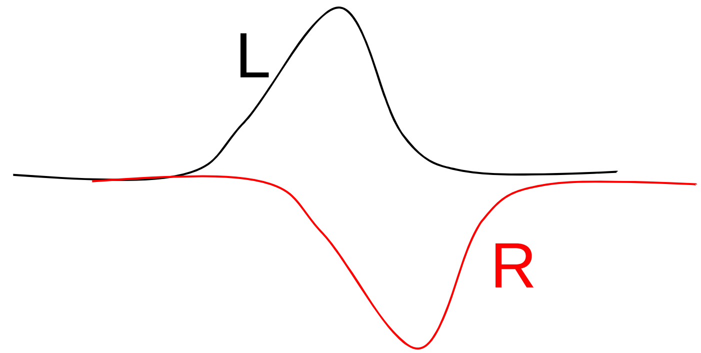


Two components of wave function, left and right, added together to make full wave function.

Write down what the final wave function looks like for the two cases (peaks/dip are same distance apart)

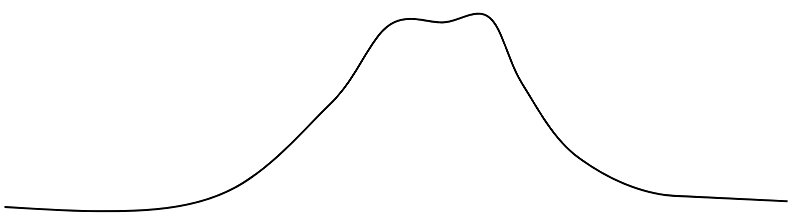
a. the two combined peaks look ~the same, both give higher wider peak.
b. look different, 1 higher and wider, 2 has two smaller peaks.
c. look different, 1 higher and wider; 2 lower peak on left, valley on right.



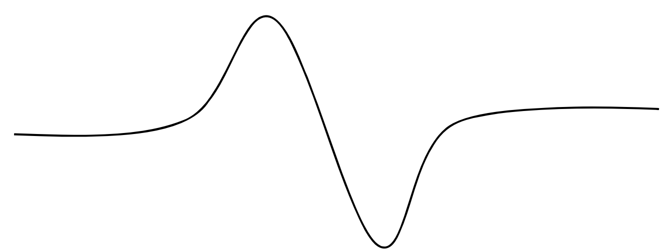


Two components of wave function, left and right, added together to make full wave function. Ans. c

1



2



What will measure for position and KE of 1 and 2?
(assume have many identical copies of wave function, do measurements bunch of times)

1. position?

2. Kinetic energy?

a. 1 has more, b. both have same, c. 2 has more

d. cannot tell

which has more momentum?

ans. 2 has more. Relates to molecular bonding energies

Superposition

- If $\Psi_1(x,t)$ and $\Psi_2(x,t)$ are both solutions to wave equation, so is $\Psi_1(x,t) + \Psi_2(x,t)$. → Superposition principle
- E.g. homework 2 – superposition of waves traveling left and right create standing wave:

$$E(x,t) = A\sin(kx - \omega t) - A\sin(kx + \omega t) = -2A\sin(kx)\sin(\omega t)$$

- We can make a "wave packet" by combining plane waves of different energies:

Comparing waves

EM Waves (light/photons):

Amplitude E = electric field

E^2 tells you probability of finding photon.

Maxwell's Equations:

$$\frac{\partial^2 E}{\partial x^2} = \frac{1}{c^2}\frac{\partial^2 E}{\partial t^2}$$

Solutions are sin/cosine waves:

$$E(x,t) = A\sin(kx - \omega t)$$
$$E(x,t) = A\cos(kx - \omega t)$$

Matter Waves (electrons, etc.):

Amplitude ψ = "wave function"

$|\psi|^2$ tells you probability of finding particle.

Schrodinger Equation:

$$-\frac{\hbar^2}{2m}\frac{\partial^2 \psi}{\partial x^2} = i\hbar\frac{\partial \psi}{\partial t}$$

Solutions are complex sine/cosine waves:

$$\psi(x,t) = Ae^{i(kx-\omega t)} =$$
$$A(\cos(kx - \omega t) + i\sin(kx - \omega t))$$

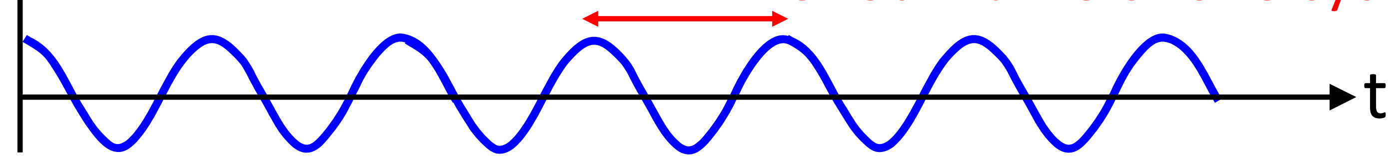
sign of $\mathcal{E}$ important when adding waves -- interference
same for ψ

a. I am very familiar and comfortable describing waves in terms wave vectors (k’s).
b. Not so familiar, would like a brief review.

Review of sinusoidal waves:

<u>Wave in time:</u> $\cos(2\pi t/T) = \cos(\omega t) = \cos(2\pi f t)$

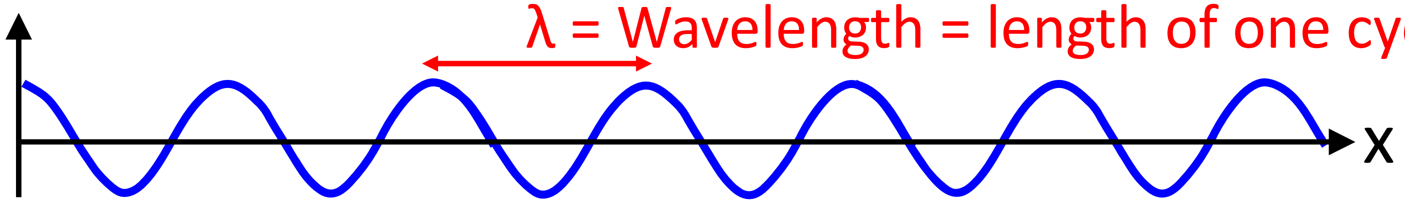
T = Period = time of one cycle



$\omega = 2\pi/T$ = angular frequency = number of radians per second

<u>Wave in space:</u> $\cos(2\pi x/\lambda) = \cos(kx)$

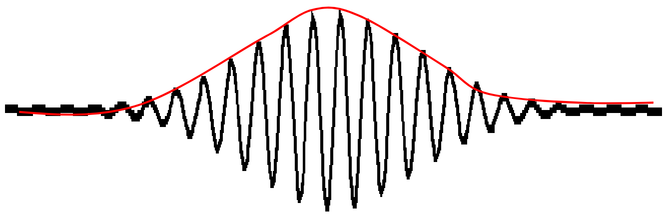
λ = Wavelength = length of one cycle



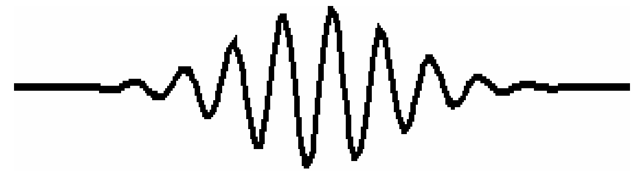
$k = 2\pi/\lambda$ = wave number = number of radians per meter

k is spatial analogue of angular frequency ω.

We use k because it's easier to write $\sin(kx)$ than $\sin(2\pi x/\lambda)$.



A



B

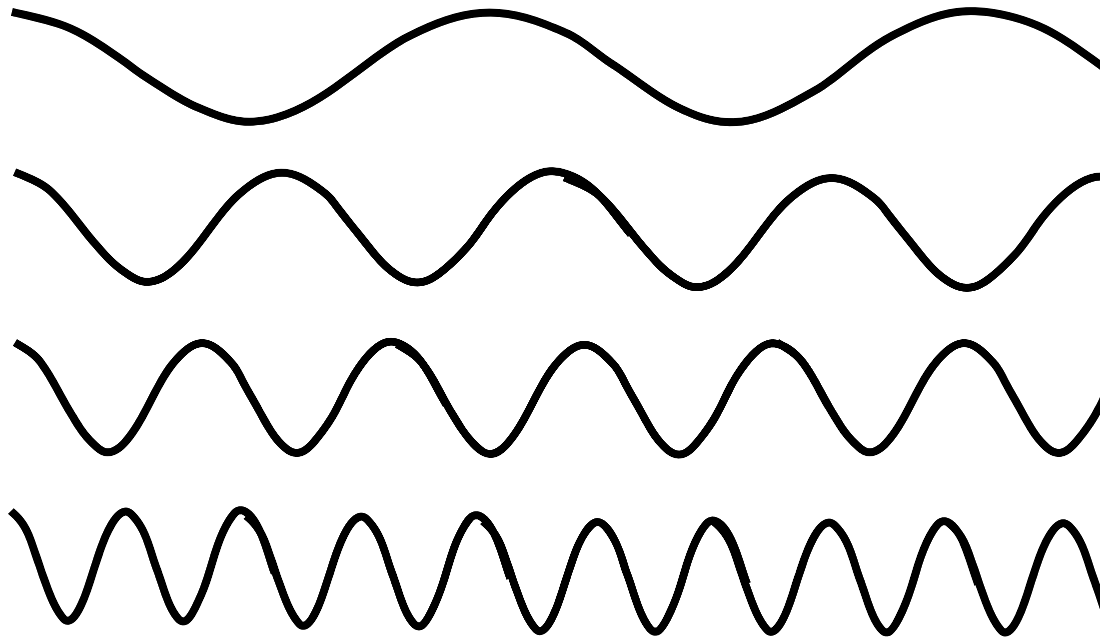
two wave packets for electron-- same envelope-- demo

a. same spread in x, same values of momentum
b. A smaller spread in x, bigger spread in p
c. A and B have same x and p values
d. A is moving slower than B, but has same spread in x.
e. A is moving faster than B, but has same spread in x.

Plane Waves

- Most general kinds of waves are plane waves (sines, cosines, complex exponentials) – extend forever in space

- $\Psi_1(x,t) = \exp(i(k_1 x - \omega_1 t))$
- $\Psi_2(x,t) = \exp(i(k_2 x - \omega_2 t))$
- $\Psi_3(x,t) = \exp(i(k_3 x - \omega_3 t))$
- $\Psi_4(x,t) = \exp(i(k_4 x - \omega_4 t))$



- etc…

Different *k*'s correspond to different energies, since p=h/λ = $\hbar$*k*

$$E = mv^2/2 = p^2/2m = h^2/2m\lambda^2 = \hbar^2 k^2/2m$$

Heisenberg Uncertainty Principle

Plane Wave: $\Psi(x,t) = A\exp(i(kx-\omega t))$

- Wavelength, momentum, energy well-defined.
- Position not well-defined: Amplitude is equal everywhere, so particle could be anywhere!

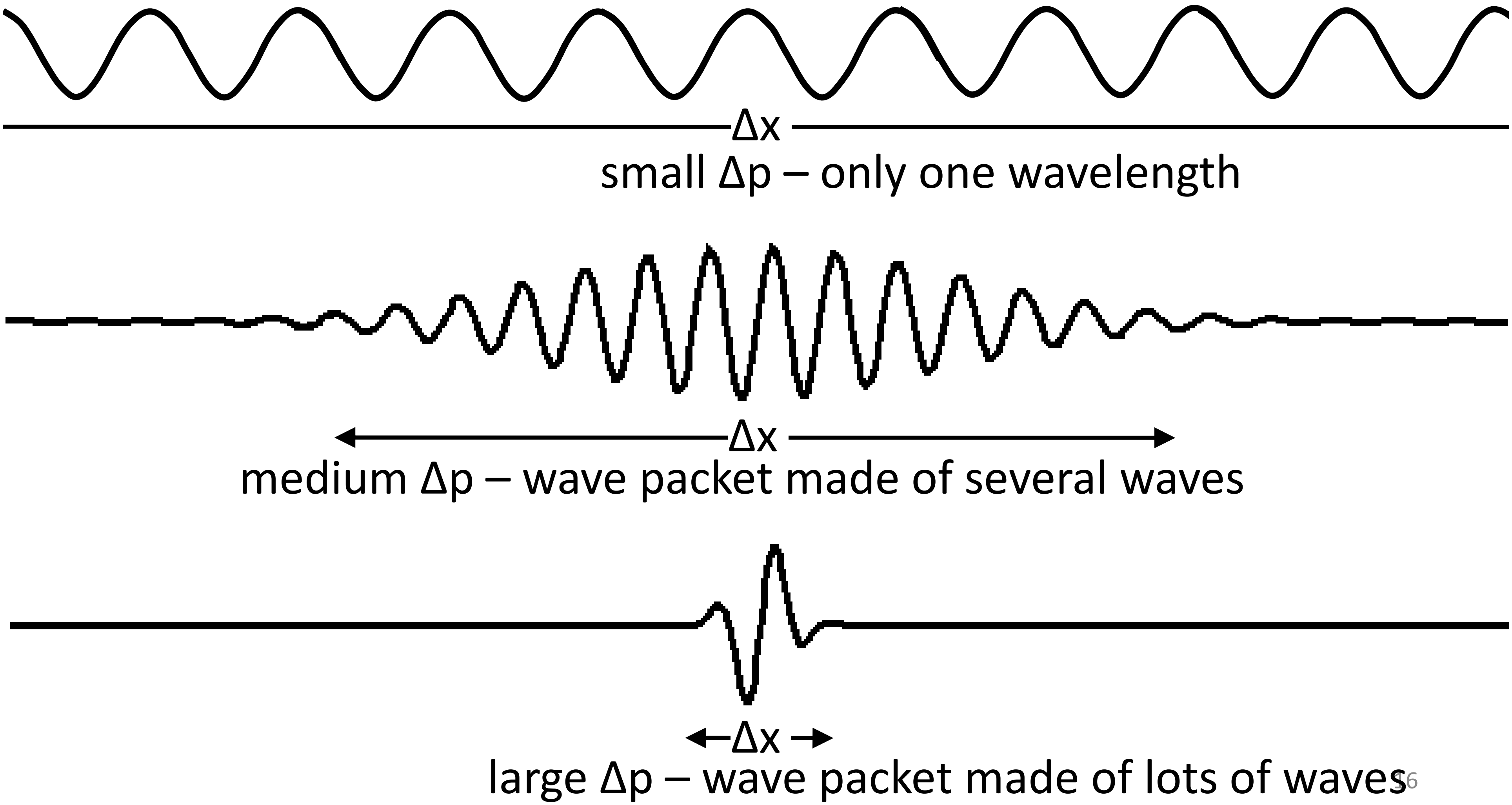
Wave Packet: $\Psi(x,t) = \Sigma_n A_n \exp(i(k_n x-\omega_n t))$

- λ, p, E not well-defined: made up of a bunch of different waves, each with a different λ,p,E
- x much better defined: amplitude only non-zero in small region of space, so particle can only be found there.

Heisenberg Uncertainty Principle

- In math: $\Delta x \Delta p \geq \hbar/2$
- In words: Position and momentum cannot both be determined completely precisely. The more precisely one is determined, the less precisely the other is determined.
- Should really be called "Heisenberg Indeterminacy Principle."
- This is weird if you think about particles, obvious if you think about waves.

Heisenberg Uncertainty Principle



Using the H. U. P. to figure out physics we do not know.

A proton is stuck inside an atomic nucleus composed of other protons and neutrons. Is size of about 10^{-14} m as Rutherford discovered.

Implies there must be some force holding it in because protons repelling. Proton is confined in some potential energy by this force.

What is the scale of the confining energy?

So about how energy required to tear nucleus apart?

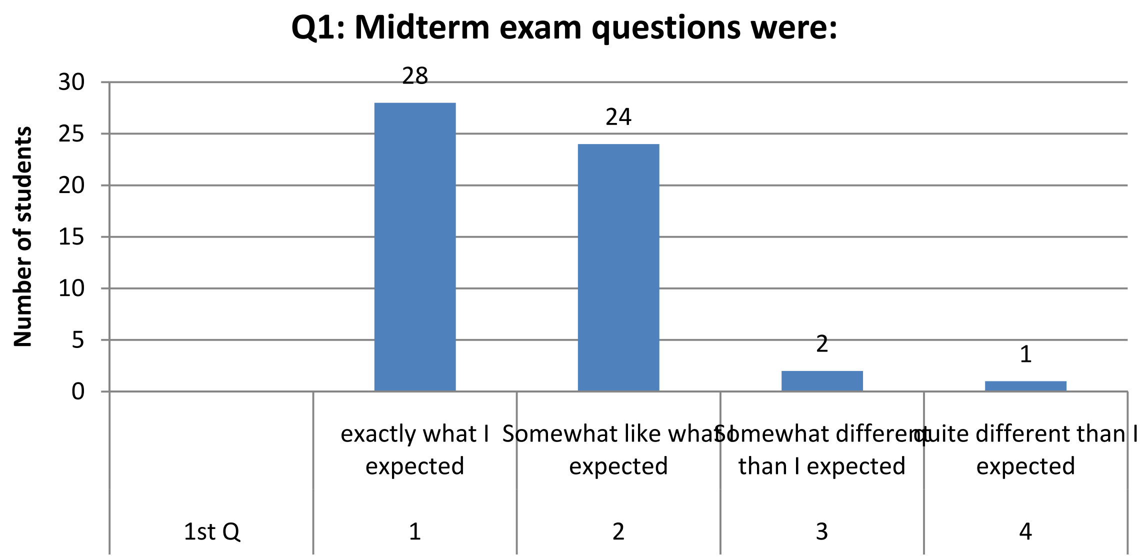
Why would energy be related to size of nucleus?

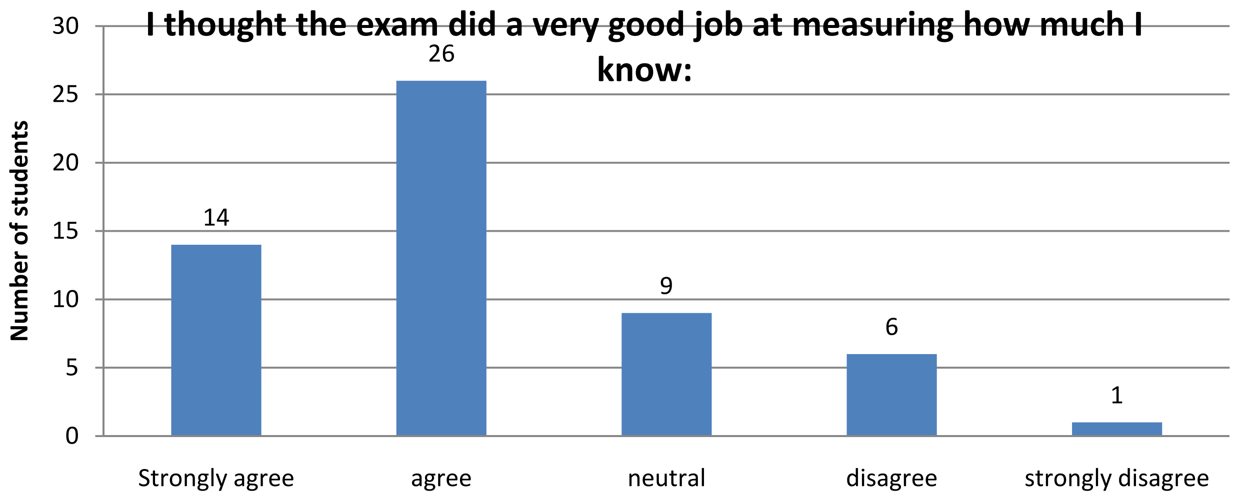
group discuss, write down, one short sentence or less,
if can come up with relation in words, try and express as eq.

to confine within small position range Δx means must have large uncertainty in momentum, $\Delta x \Delta p \geq h/2$.
Means have large range of momentum components in wave function, so large momentum in wave function.
Large momentum means large kinetic energy.
How express as equation?

$p \sim \Delta p \sim h/2\Delta x$, where $\Delta x = 10^{-14}$ m
$KE = p^2/2M_p = (h/2 \times 10^{-14} \text{ m})^2/2M_p \ = ?$

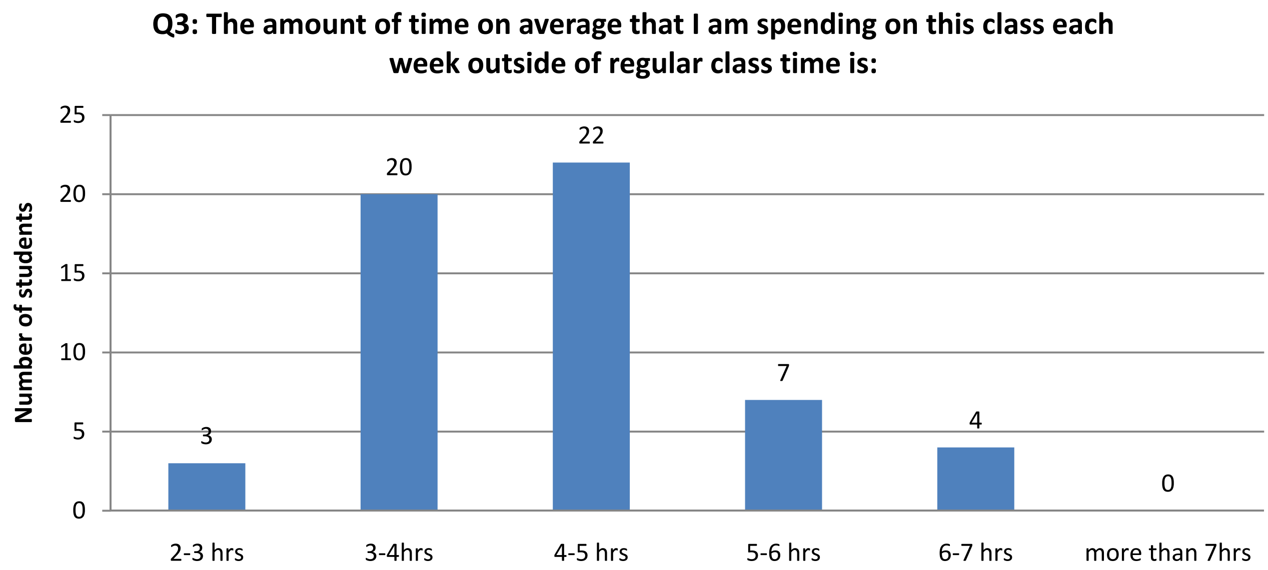
2 M eV = 3.2×10^{-13} J - - typical energy for alpha particle in alpha decay

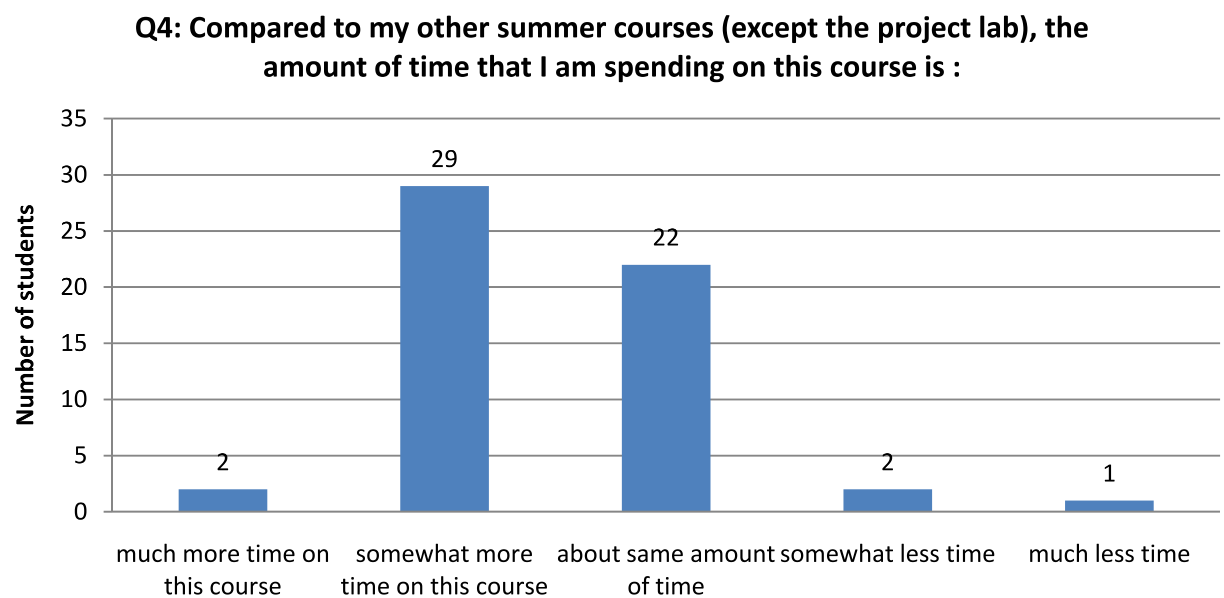


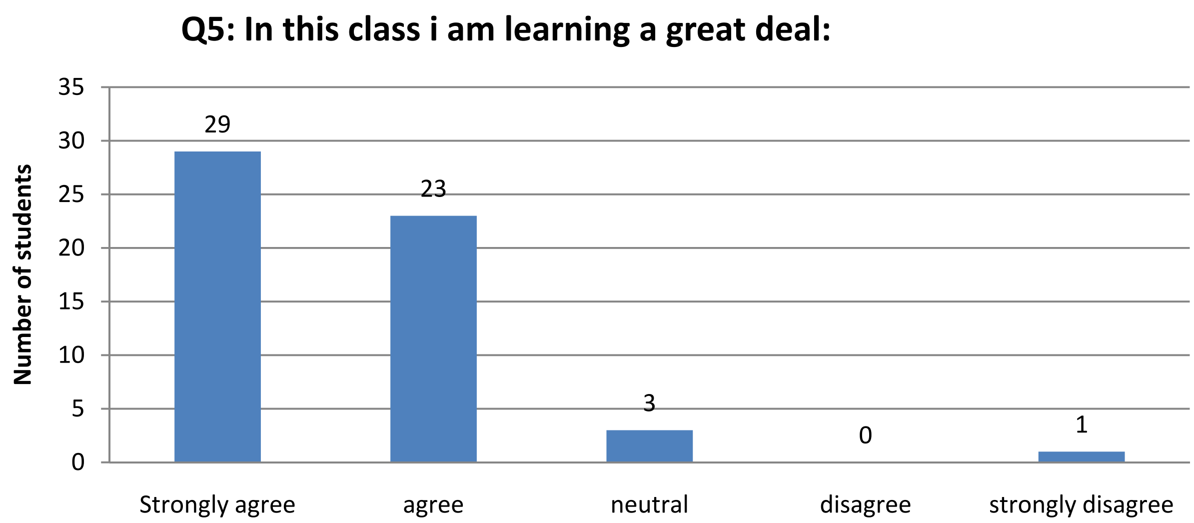


Ph. El. Eff. question

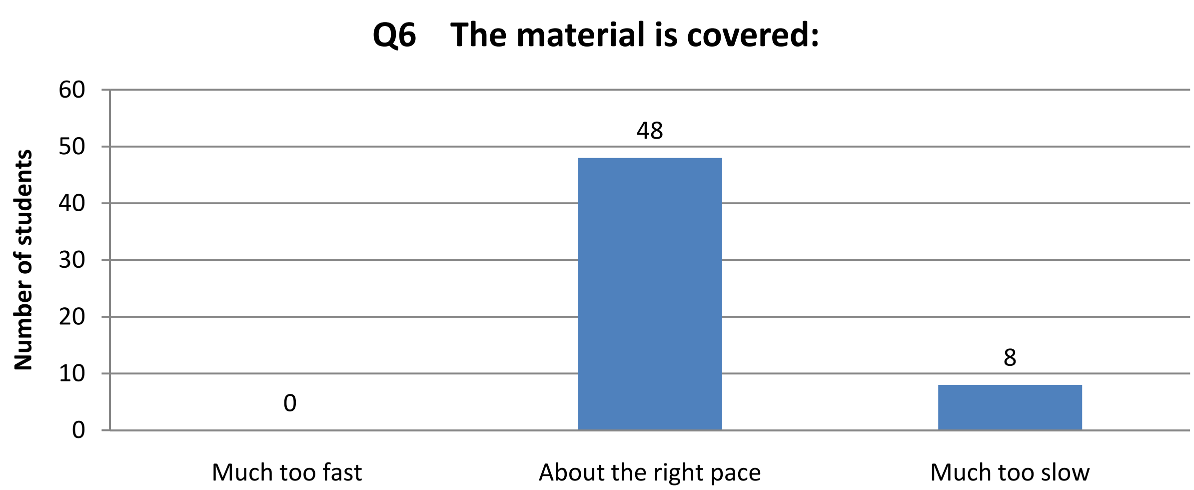
I was unhappy that number of people got < 70% on individual exam. Means did not learn >30% of what should.


Q3: The amount of time on average that I am spending on this class each week outside of regular class time is:
Number of students
25
20
15
10
5
0
3
20
22
7
4
0
2-3 hrs
3-4hrs
4-5 hrs
5-6 hrs
6-7 hrs
more than 7hrs


Q4: Compared to my other summer courses (except the project lab), the amount of time that I am spending on this course is :
Number of students
35
30
25
20
15
10
5
0
2
29
22
2
1
much more time on this course
somewhat more time on this course
about same amount of time
somewhat less time
much less time

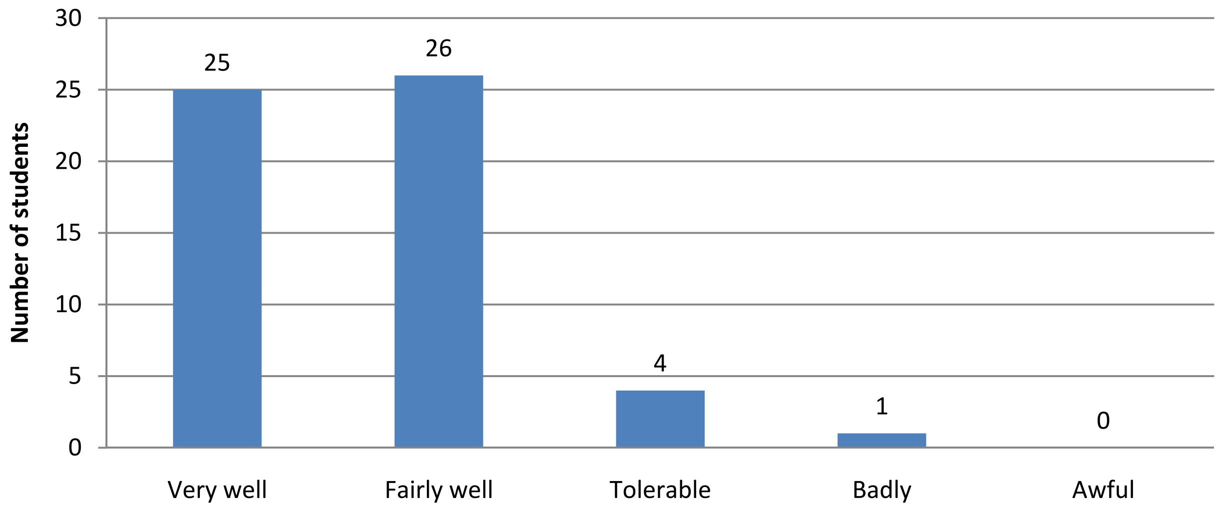


Q5: In this class i am learning a great deal:
Number of students
35
30
25
20
15
10
5
0
29
23
3
0
1
Strongly agree
agree
neutral
disagree
strongly disagree



Q6 The material is covered:
Number of students
60
50
40
30
20
10
0
0
48
8
Much too fast
About the right pace
Much too slow

Q7 In class my group is working together:



“Things that help my learning”
Every part of course mentioned by multiple people.

Things that could be better (nothing very common)--

1. most common-- group seating arrangement.

a. Talking within group.

b. Hearing student questions and comments from front if seated in back.

response-- a. classroom setup inherent problem, searched entire campus for room, these were best we could get.

b. will rearrange group locations-- new location given out as come into class Wednesday.

2. Group discussions. A few things about people dominating group discussion,
not having broad participation, getting bogged down in two individuals arguing.

Response-- talk about in minute

3. a few-- Break up tutorial and give more feedback.
response-- done, starting last Friday.

4. Harder clicker questions.
Response-- I completely agree! Working on it!
Will also speed up course, but only slightly.

5. one person-- wants me to have office hours.
Response-- I always have had them. Are posted on website, 4-5 PM Mondays.

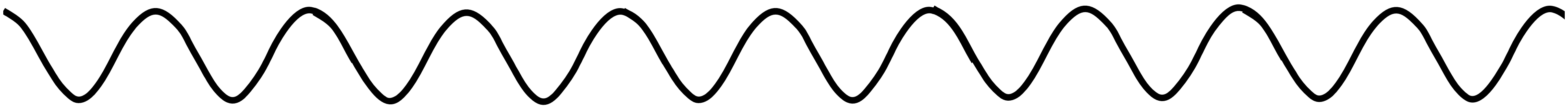
6. More homework. More practice problems.
Response-- book and workbook lots of practice problems, no additional HW problems, would exceed agreed upon workload.

Group interactions-- skills learned useful for rest of life.
a few observations about the groups
1) comparing individual and group exam scores
a. 27% of men got higher individual score than group score
b. 66% of women got higher individual than group score
c. in all groups where there were indications of someone dominating the discussion, that person was not highest scoring individual in the group.

<u>lesson</u>-- need to be making more effort to involve all members of group in discussion and getting benefit of all input. Periodically stop, go around and get input from everyone in group. Particularly good to do when argument between two going on very long.
Discussion?

Plane Waves vs. Wave Packets

Plane Wave: $\Psi(x,t) = A\exp(i(kx-\omega t))$



Wave Packet: $\Psi(x,t) = \Sigma_n A_n \exp(i(k_n x-\omega_n t))$

For which type of wave are position x and momentum p most well-defined?

A. p most well-defined for plane wave, x most well-defined for wave packet.

B. x most well-defined for plane wave, p most well-defined for wave packet.

C. p most well-defined for plane wave, x equally well-defined for both.

D. x most well-defined for wave packet, p most well-defined for both.

E. p and x equally well-defined for both.