

## Math 307: homework problems 8

1. Using the power method, find the eigenvalues closest to  $-6$  and closest to  $1$  for the matrix:

$$A = \begin{bmatrix} 1 & 7 & -11 & 2 & 5 \\ 0 & 1 & 4 & 8 & -2 \\ 0 & 1 & 0 & 1 & 0 \\ 2 & 0 & 2 & 4 & 5 \\ 7 & 8 & 2 & 3 & 0 \end{bmatrix}$$

Write down the MATLAB/Octave commands you used to find your answer.

2. Find the solution of the recursion relation

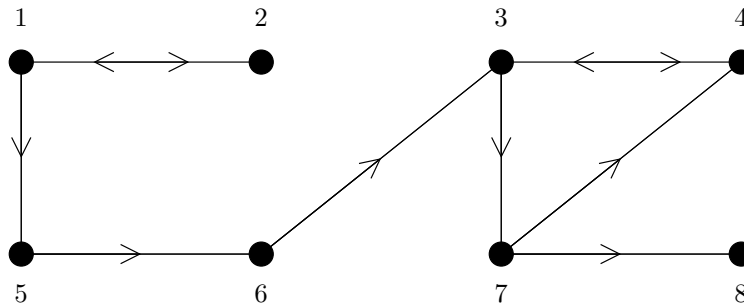
$$x_{n+2} - 2x_{n+1} + 2x_n = 0$$

with initial conditions  $x_0 = 1$  and  $x_1 = 1$ .

3. Using MATLAB/Octave or otherwise, find  $x_{30}$ ,  $x_{31}$  and  $x_{32}$  for the recursion relation defined by  $x_0 = 0$ ,  $x_1 = 1$  and  $x_2 = 2$  and

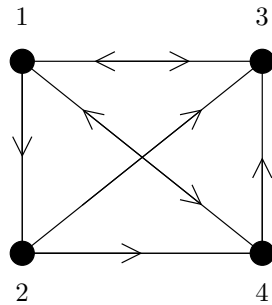
$$x_{n+1} = x_n - 2x_{n-1} + x_{n-2}$$

4. Suppose that there is a fixed population of cola drinkers each with a favourite among Coke, Pepsi and ThumbsUp. Every month 3% of the Coke drinkers switch to Pepsi while 5% switch to ThumbsUp. Every month 2% of the Pepsi drinker switch to Coke while 3% switch to ThumbsUp. Every month 1% of the ThumbsUp drinkers switch to Coke and 1% switch to Pepsi. What are the eventual market shares of these drinks?
5. A flea hops randomly on vertices of a triangle, hopping to each of the other vertices with equal probability (never remaining at the same vertex). The flea starts at vertex 1. What is the probability that the flea is at vertex 1 again after  $n$  hops?
6. Show that the product of two  $n \times n$  stochastic matrices is also stochastic.
7. Find the rank (using PageRank) for each site with  $\alpha = 1$  and  $0.85$  for an internet that is linked in the following way



You can use `eig` to find the relevant eigenvector, but for  $\alpha = 0.85$ , check that the power method gives the same answer (give the MATLAB/Octave commands you used for the power method).

8. Consider the following internet:



If  $\alpha = 1$ , show that the rankings (using PageRank) of the pages are such that page 1 is highest.

The owners of page 3 are furious that their page doesn't rank highest. They try to increase the rank of page 3 by creating a new page, page 5, to which page 3 links and which links to page 3. Does this increase the rank of page 3 above that of page 1 (with  $\alpha = 1$ )?

9. (Bonus) Prove that for an internet, the rankings (using PageRank) of a page having no links to it from any other page is always zero assuming  $\alpha = 1$ .