5 Minute Mathematics Break.

Last lecture we found the general solution of

$$y'' - 6y' + 9y = 0.$$

It was

$$y(t) = c_1 e^{3t} + c_2 t e^{3t}.$$

Show that c_1 and c_2 can be chosen uniquely to satisfy any initial value problem

$$y(t_0) = a$$
$$y'(t_0) = b.$$

(any t_0, a, b).

Ex Find a particular solution of
$$y'' - 3y' - 4y = 2\sin t$$

Ex Find a particular solution of
$$y'' - 3y' - 4y = te^{2t}$$

Ex Find a particular solution of
$$y'' - 3y' - 4y = te^{2t} + 2\sin t.$$

Ex Find the general solution of
$$y'' + 5y' + 4y = e^{-4t}$$

Ex Find the solution of the IVP
$$y'' + 4y' + 4y = e^{-2t}$$
with $y(0) = 0$ and $y'(0) = 0$.

Ex Find the solution of the IVP
$$y'' + 4y = \sin \omega t \quad \text{with } y(0) = 1 \text{ and } y'(0) = 0.$$

For which values of ω does the IVP have solutions that become unbounded as $t \to \infty$.

5 Minute Mathematics Break.

For the forcing terms $g(t)$ below, what trial functions would you use for the MUC? The homogeneous solutions are given.

1) $y_c = c_1 \sin t + c_2 \cos t$, $g(t) = e^t$

2) $y_c = c_1 + c_2 e^{2t}$, $g(t) = 1 + t^2$.

3) $y_c = e^{2t} + te^{2t}$, $g(t) = 5e^{2t}$

4) $y_c = c_1 \sin t + c_2 \cos t$, $g(t) = \sin t \, e^{-t}$

5) $y_c = c_1 e^t + c_2 e^{-t}$, $g(t) = 1 + 4t^3 - t\sin 2t$.

Ex $y'' - 6y' + 9y = 0$
has a homogeneous soln $y_1 = e^{3t}$.
Use Reduction of order to find a second homogeneous soln.

Ex Find the general solution of

$$t^2 y'' - 2ty' + 2y = 4t^2$$

given that $y_1(t) = t$ is a solution of the homogeneous problem.

Ex Consider $t^2 y'' - 2y = 3t^2 - 1$.

a) Show that the homogeneous equation has a solution of the form $y = t^n$ (n to be determined).

b) Find the general soln.

Ex Find the general solution of
$y'' + y = \sin t$
given that $y_1(t) = \sin t$ is a soln of the problem. Use the method of reduction of order.